ZNAMENNY SCALE – FAIT ACCOMPLI?*

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Abstract

The author addresses one of the most sensitive topics of Znamenny chant: its scale. He tries to restore the "burned bridges" between ideographic and staff-notation of the Chant as he redefines and essentially generalizes the concept of scale as such. The possibility to artificially construct ad hoc many scales sounding sometimes very similar to the scale suggested by the modern staff-notation is a serious argument to regard the "staff-notation based" deciphering from the 17th century (dvoeznamenniki) a pure game of chance. The constructed scales, presented in the paper, are different to "keyboard diatonica" and from one another and are never subject to the unified theorizing (unified nomenclature of degrees, etc). Critically commented is the practice to uncontrollably use the trivial pitch-symbols for deciphering, which ipso facto makes the probabilistic steps of unknown scales look as the ill-founded deviations from the diatonic scale steps, which are currently in use in the common musical education. This practice hinders the chance to acknowledge the right of the remote musical culture to rest on foundations that can be formulated both positively and explicitly, all the more so, as the usage of paleographic signs looks rather consistent. The resemblances and differences between musical cultures may be treated more liberally since no scale is seen a norm. The author is based on the writings of Russian musicologist and organologist Felix Raudonikas.

***Keywords*:** Znamenny Chant, Deciphering, Paleography, Solmisation, Guidonian hexachords, Temperament, Tuning, Diatonics, Old-believers, Emergence, Mode, Glas, Aristoxenos, Tone, Accidentals, Metavola, Modulation, Mese, Paramese, Division of octave, Pomety, Cinnabar letters, Dvoeznamenniki.

THE PROBLEM

Historical success of first decryptions of znamennaya notation has been feasible due to the so-called dvoeznamenniki, kind of bilinguals – monuments, where the same repertory was written parallel in Znamenny and staff notation. Affinity between sistema teleion micron – inherent for Guidonian hexachords – and recommendations, contained in Russian singing manuals of that time, gave occasion to certain conjectures concerning the structure of Znamenny scale.[1]

* The thrust of this paper and its general approach have been inspired by the Saint-Petersburger musicologist and organologist Felix Raudonikas, who held the seminar "Musical syntax" at the State University of Saint-Petersburg in the academic year 2001/02. The computational methods, used in this article are presented in: F. Raudonikas, *Pifagoreyskaya sistema muzykal'nyh tonov*, Chasti 1, 2, Seriya "Problemy muzykoznaniya", Vypuski 2 and 5, Leningrad, 1988 and F. Raudonikas, *Sintaksis relativnogo dvuhmernogo modusa*, in: "Problemy muzykoznaniya", Vypusk 8, Saint-Petersburg, 1996. In the tables author also used the data (frequencies) from CD-supplement to: Ravdonikas F. V., *Muzykal'nyj sintaksis*, 2-e izdaniye. Saint-Petersburg, Earlymusic Publishing House, 2007.

[1] 1. M. Brazhnikov, *Drevnerusskaya teoriya muzyki*, Moskva, 1972, P. 308: "System of solmisation has found its place in ancient Russian theoretical znamenny azbuka's and to significant extent intertwisted with the theory of Znamenny chant not only because of collision of two systems (Russian and occidental – A. Y.) but also because the hexachord-system being in fact close to the system of Russian soglasiyas (accordances – technical term of Znamenny chant – A. Y.) and having been perceived by the Russian singers as something sufficiently familiar and not contradictory to the customary theoretical statements". ["Сольмизационная система нашла место в древнерусских теоретических знаменных азбуках и в значительной степени переплелась с теорией знаменного пения не только в силу столкновения двух музыкальных систем, но и потому что система гексахордов была по существу близка системе русских согласий и была воспринята русскими певцами как нечто, уже в достаточной степени знакомое и не противоречащее привычным теоретическим положениям"]. The notion of znamenny scale (obyhodny zvukoriad, znemenny zvukoriad) is hard to trace back up its origin. M. Brazhnikov (*op. cit.*, p. 271): "Gorovoskhodny holm's (special diagrams showing the scale – A. Y.) are however more spectacular and they are designed directly for demonstration of the scale, whereas pomety's (kind of litterae significativae – A. Y.) are only nollens-vollens being "stacked" in the scale, and function of each other

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Fig. 1 – The supposed Znamenny scale.

Staff-notation rudely "disciplines" Znamenny melodies; this fact has been underlined by many researchers. The results of ethnographical field research also justify this opinion. Yet staff-notation doesn't make Znamenny melodies sound "Gregorian". That lessens the responsibility of staff-notation for the fate of Znamenny Chant, that, according to opinion of many researchers, couldn't survive the collision with Western musical system.

Trying to explain the existing discrepancy between staff notation and Znamenny melodies some researchers recur to the concept of "temperament", which is treated as a symbol of the staff-notation "artificiality". According to the opinion of one of the most prominent scholars M. Brazhnikov Znamenny melody is characterized by natural tuning, which also characterizes folk songs.[2] Yet, the definition of naturalness is challenging – as opposed to such a formal thing as scale, the naturalness can be neither presented nor verified.

Znamenny chant is being ascribed a Diatonic gender. Yet, there are many historical definitions and "species" of the diatonic scale (Ptolemeus, Archites etc), and in case of Znamenny chant this scale is still not specified. A. N. Miasoedov (O garmonii v russkoy muzyke. 1998) correctly attests that "the diatonicity of obihodny scale has been never and by nobody questioned".[3] However, it has been never explicitly postulated as well. It looks like as if a title of "diatonicity" were granted to Znamenny chant as "right to citizenship" in exchange for a promise never to be interested in the "rights of citizen". This adherence to the unvoiced conventions makes Znamenny chant even more dependent on the staff-notation, than the Occident chant is.

This dependence results in fact, that graphical appearance of the musical text and tuning are brought in research so close to one another that they are felt nearly interchangeable – Western five –line system is declared to have destroyed Znamenny chant because this system was well-tempered.[4] But this notation has nothing to do with temperament. It can suit with the same success Pythagorean, Pretorius, Werkmeister, or mean-tone.

The correct idea of heterologous, or xenogenic character of staff notation in regard to Znamenny chant shattered the trust in *any* analytic (=non-stenographic) notation to be possible to render Znamenny chant more accurately as staff notation does, and demotivated the interest to the theoretical foundations of the scale. (Few lonely figures like Yuri Arnold are exceptions confirming the general rule.[5])

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pometa exists on its own. In general it is symptomatic, that although the information on 12 (or may be more) "degrees" can be obtained from azbuka's, the notion of the scale itself can be defined by no direct way, but only in oblique manner". ["Горовосходные холмы, однако, более наглядны и предназначены непосредственно для показа звукоряда, тогда как пометы только волей-неволей складываются в звукоряд и назначение их самодовлеюще для каждой в отдельности. Вообще же показательно, что из азбук можно почерпнуть сведения о 12 (а может быть и больше) «степенях», но самое понятие церковного звукоряда в них определяется не каким-нибудь прямым способом, а лишь косвенными средствами"].

[2] Brazhnikov, *op. cit.*, p. 386: "What was the tuning of the ancient Russian chants from the earliest times up to its collision with the West-European system? This can be the only answer on this question – certainly it was natural, that tuning, that formed a base for creating and performing of Russian folk song". ["Каким был строй древнерусских распевов с древнейших времен и до его столкновения с западноевропейской системой? Ответ может быть только один – конечно натуральным, тем самым, на основе которого создавалась и существовала русская народная песня"]. *Ibidem*, p. 334: "One shan't forget that znamenny chants have been violently "dragged" to the tempered tuning with its major, minor and diminished triads, whereas it is the natural tuning that forms a base both of the chant and of folk song". ["Не следует забывать, что знаменные напевы в силу необходимости оказались насильственно притянутыми к темперированному строю с его мажорными, минорными уменьшенными трезвучиями, тогда как в основе его, как и народной песни, его лежит натуральный строй"].

It is not clear whether the researcher signs the rights of Znamenny chant over the triads away to the benefit of tempered tuning or he means tempered triads. In both cases the passage implies the only possible (=tempered) way of harmonization.

[3] A. N. Miasoedov, *O garmonii v russkoy muzyke*, Moskva, 1998. The work contains very important considerations about the notion of "diatonica" and problems concerning "intensional" definition of the latter.

[4] Brazhnikov, *op. cit.*, p. 386: "The main peculiarity of five-lines representation of Znamenny chants is, that by that way of record the chants are nollens-vollens fall into dependence of tempered tuning, although the singers, who performed the chants according to five-line-notes might not always adhere this tuning". ["Главная особенность пятилинейного изложения знаменных напевов состоит в том, что при этом способе записи они волей-неволей оказываются в зависимости от темперированного строя, хотя, быть может, певцы – исполнители по пятилинейным нотам не всегда точно его придерживались"]. *Ibidem*, p. 385. "It won't be correct to speak about five-lines-notation without bearing in mind, that each system of notation can exist only, if it expresses and renders completely the musical system from which it originates together with its theoretical content". ["Нельзя говорить о пятилинейной нотации, забывая при этом, что любая система нотной записи может существовать только в том случае, если она выражает и передает полностью музыкальную систему, ее породившую, вместе с ее теоретическим содержанием"].

[5] Juri Arnold, *Die alten Kirchenmodi, historisch und akustisch entwickelt*, Leipzig, 1878; id., *Teoriya drevnerusskogo tserkovnogo i narodnogo peniya*, Moskwa, 1880.

Such resignation leads to strange situation: until now the "Guidonian-notated" solution is respected as a checkpoint after which the paleographical monument can be flagged as "deciphered"! Until now the alternatives "e vs f" are decisive for correct attribution of documented melodic formula or detecting it in the monument. These alternatives are also responsible for presenting the structural units of the scale – such as "soglasiyas" ("agreements", so readily identified with halves of Guidonian hexachordia) – or at least endowing such terms with material connotations. Predominance of melodic interpretation of the chant, mainly adopted nowadays, doesn't help to overcome textual specificity of trivial staff notation. What brings the distancing from the alien staff-representation of the Chant if in the long run one has to return to such kind of problems – e vs f , or b vs h, that remind on the famous "una nota super la semper est canenda fa"?! Needless to say, that these alternatives presuppose unvoiced convention about the only one possible distribution of "correct" tones and semitones in the scale according to some perfect yet obscure rule.

On the other hand, the audio-records of modern old-believers are often notated in-staff with accidentals.[6] This betrays dependence of decipherers on western norms rather than proves independence of Znamenny chant from any possible rationalization. If the accidentals are *deviations* – why the norm (which they deviate from) is being felt *so alien* to Znamenny chant, if, otherwise, they reflect some *modal content* (e.g. transposition) – what remains of the integrity of the back-bone of the chant – thesaurus of rigid (up to its intervallic) melodic formulas? If the degrees (or *some* degrees) are variable par excellence – what circumstances are responsible for the range of these variations? Can these circumstances vary in their turn or they are steady? And finally – why can the melody be recognized as such in spite of significant variations of the pitch of the scale degrees? Why was it *still possible* to render Znamenny melodies in European staff notation in spite of all theoretical shortcomings and impoverishment of melodic language?

EXPECTANCY

Znamenny melodies don't sound Gregorian, in spite of the fact that they are both diatonic. Scottish melodies don't sound Chinese in spite of the pentatonic character they share. Why? Trying to answer this question one can recur to the concept of the mode. The mode as an instance intermediating between scale and melody is sometimes ascribed selectivity in regard to scale (e.g. Strebetoene, notes sensibles, tendency pitches, or their counterparts – Attraktions-toene, estotes, ustoi). The alternative in the mode definition (generalized tune vs particularized scale, see the article "Mode" by H. Powers et al. in The New Grove Dictionary of Music and Musicians) leads the decipherer into the methodological stone-sack. While trying to sublate this "melody-scale" opposition we tend to describe mode in terms of what it does, rather than what it is:

The mode establishes some scale or scales (as specimen or specimens) and at the same time it utilizes the scale as general substrate for potential states of musical form.

By this angle of view the mentioned selectivity in regard to scale would mean the choice between different species of some different scales-specimens rather than the choice between Attraktionstoene or specific intervals of some paradigmatic scale containing or representing all others. Scales – as ensembles of their degrees – then would act as different *measures* of raising and lowering of pitch, offering different *agreements* of the parts of melody, thus endowing the ensemble of the frequencies with the features of the semantic whole.

Recurring to the philosophical concept of emergence,[7] one can reformulate the above-mentioned ambiguous definition of the mode more positively. Mode is the whole of its degrees, *i.e.* – more than just their conglomerate. As the whole it may contain features that its parts don't contain. The character of this

[6] One of the greatest tributes in the research of modern old-believers singing has been made by T.F. Vladyshevskaya (*Muzykal'naya kul'tura Drevney Rusi*, 2006). The deviation from the "obyhodny zvukoriad", observed by the modern old-believers, are explained here with the help of transposition of the whole scale. The interval of transposition is major second.

[7] "The emergent is unlike its components insofar as these are incommensurable, and it cannot be reduced to their sum or their difference" (G. H. Lewes, *Problems of mind and life*, 1875, p. 412).

"Although strong emergence is logically possible, it is uncomfortably like magic. How does an irreducible but supervenient downward causal power arise, since by definition it cannot be due to the aggregation of the micro-level potentialities? Such causal powers would be quite unlike anything within our scientific ken. This not only indicates how they will discomfort reasonable forms of materialism. Their mysteriousness will only heighten the traditional worry that emergence entails illegitimately getting something from nothing" (Bedau Mark A., *Weak emergence*, 1997).

whole may persist in spite of variations of its parts (e.g. of degrees). But it also may suddenly change, if the variations run up to some critical value.

In order to make the connections of the parts of the whole more traceable, one needs the multidimensional representation of the mode. In fact, one-dimensional representation can be given to *any* combination of tones, also to a random one. The two-dimensional representation of mode can be given only to the regular combinations with verifiable features. This demand of verifiability well corresponds with both the restrictions and hierarchy, contained in the above quoted definition of mode ("To attribute mode to a musical item implies some hierarchy of pitch relationships, or some restriction on pitch successions"). The most familiar regular combinations are the diatonic modes of Aristoxenos.[8] First – one knows the rule how they are modeled, second – the number of their degrees is fixed.[9] So the modes are consistent and finite. Each of them can be rearranged into continuous sequence of perfect fifths (or perfect fourths). That's why the modes can be given as plane convex two-dimensional figures.

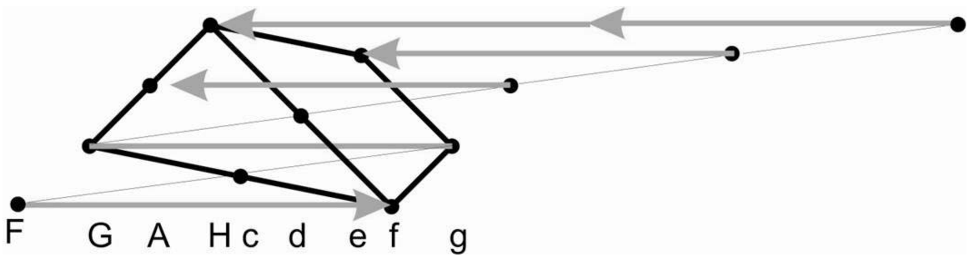

Fig. 2 – "Mixolydian".

Fig. 2 shows an example of such representation. Blue line without arrows represents the ambit of the mode (in this case – octave). Both ends of it exemplify "root" or "tonica". In the context of our discussion the "tonica" is treated more like an equivalence-relation: octave-transfers of tonica are tonicas, if the scale is "octave-periodical"; fourth-transfers of tonica are tonicas, if the scale is "fourth-periodical". Thin grey line (like between F and g) represents sequence of fifths. Horizontal lines with arrows are octave-transfers which bring the obtained degrees back into the framework of an ambit (in this particular case – in the boundaries of octave). Four fifths-steps are above "tonica", two fifths-steps are below it. Fig. 3 is an inversion of Fig. 2. Throughout the article we use German symbol ***H*** instead of ***B*** (natural), and German symbol ***b*** instead of ***b-flat.***

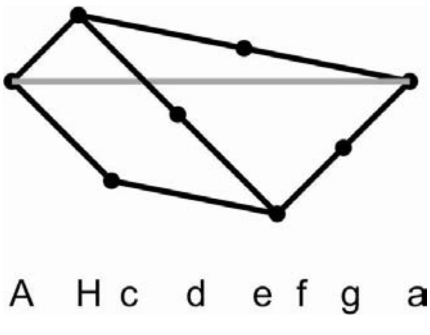

Fig. 3 – "Aeolian".

On Fig. 3 two fifths-steps are above tonica, four steps are below, the picture of Aeolian can be obtained by rotating the picture of Mixolydian to 180 degrees, and hence we speak of inversion. Both figures

[8] Aristoksen, *Elementy garmoniki*, perevod i primechaniya V. G. Tsypina, Moskva, 1997, II.

[9] "It is useful to have definitions that contain the way of generating of the object, or, in any case, if such a way is absent, the constitution, *i.e.* rule (*modus*) that makes obvoius either the repeatability or the possibility of an object" (Leibniz, *De synthesi et analysi universali seu arte inveniendi et judicandi*, in: *Sämtliche Schriften und Briefe*, Ser. 6, vol. 4, pt. A, Akademie Verlag, Berlin, 1999. Translation in English from: Leibniz, *Sochineniya v 4 tomah*, perevod s latinskogo G.G. Mayorova, Moskva, 1984, Tom 3, pp. 117–118, according to the edition of Gerhardt [VII 292-298]): "Полезно иметь определения, в которых содержится способ порождения предмета, или, во всяком случае, если этого нет, конституирование, т.е. правило (Modus) благодаря которому становится очевидной или воспроизводимость, или возможность предмета".

exemplify the distribution of steps of fifths – the ascending fifths are above the horizontal line of tonica, the descending fifths – below it. The octave-transfers (the arrows) bring the whole sequence back into the framework of the octave, specified by given position of tonic.

The unbeaten advantage of the usual staff-notation – its plausible orthography – has to do with the above considered regularities. Once the position of one mode is defined – the positions of all others are simple to calculate. This link between nomenclature of degrees and nomenclature of scales (both are merely numerations) is a result of two-dimensional presentation. The progressive evolving of the scale, *i.e.* progressive obtaining of new positions and transferring them octave-up or octave-down – will give an infinite regular plane point-system. All the "congruence motions" of this plane (= motions that let the plane coincide with itself, *i.e.* leave it as whole unchanged) – and only these – make staff notation consistent. The motions of the plane that obtain the next position in the scale and the motions that register the trans-position of the mode are essentially the same motions. That comments the link between Groove-definition of the mode and our definition: "hierarchy of pitch relationships" or motions *establishing* the scale and "some restriction on pitch successions" or motions *utilizing* the scale (perhaps in form of melodic models) are the same motions. That's why it is so easy in the European notes to discern between different positions of the same mode and to discern between different modes in the same position. That's why it is so complicated to discern between these two ways of discerning.

The trivial transposition on the one hand doesn't change the internal intervallic relationships of the scale, on the other hand – it always "computes" the number of motions – E-dur is not just "more remote" from C-dur than G-dur, but exactly "four times more remote". That is, properly speaking, the main ingenuity of notation – "altered" degree doesn't so much lower or raise a "natural" one – it much more *abolishes* the "natural" one and changes position of the whole system. The problem of modal nomenclature is solved automatically by the orthography of notation. Let us suppose that there are only 7 "natural" degrees and their octave-transfers. Once the position of "Ionian" mode (5 fifths up 1 fifth down the tonic) is given, the "Phrygian" mode (5 fifths down 1 fifth up), can be situated only 2 steps higher or 6 steps lower – no other variants (in case we are not interested in further evolving of the system = obtaining the new degrees), see Fig. 4.

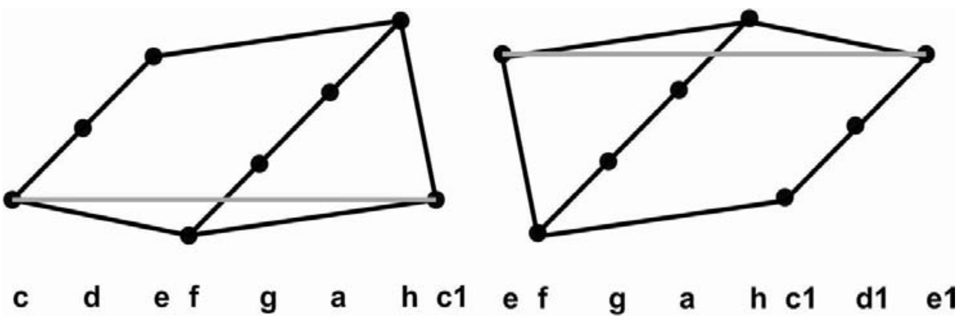

Fig. 4

The other motions (=using the alterated degrees, or, better to say, their altered symbols) are registered unequivocally by the accidentals. This shows the inherent codependence between *what* the mode is and *where* it is. That makes the speech about music very comfortable – at least in relevant contexts – and behavior of music more or less predictable.

When leaving the framework of ecclesiastical (in occidental sense) modes the researcher looses this excellent instrument of control. Imagine some melody, where the transposition "two major seconds up" is felt more "akin" to the original position, than the usual transposition perfect-fifth-up. The notation will "punish" this intuition with 3 additional sharps.

The inadequacy of staff notation in case of "non-European" material has given more occasions to anathemata than to studies. In how far does the "alien" material disorganize trivial staff notation? Answering this question one may consider the "naпve" decryptions as distortion mirrors which however sometimes allow restoring the objects. The connection between registering and distortion is deep. So, the distorted image may be more reliable than reprint – it doesn't promise too much.

Let us suppose that "key signatures" for chromatic modes are look so, see Fig. 5.

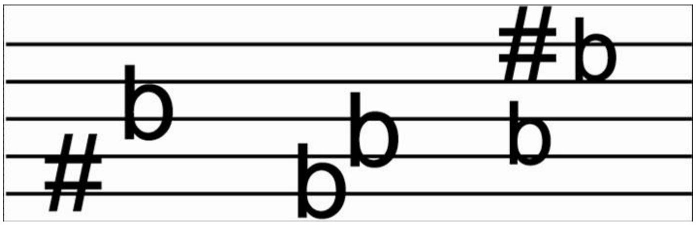

Fig. 5

They reflect some "non-European" musical reality. If the modes are rendered properly (it is to remember, that the degree with sharp or flat *abolishes* the natural one) – they can't be reorganized into continuous sequence of any interval. The next relatives of the "mode" can't be "localized". The attempts to transpose it would lead to very complicated behavior of key signatures. It is not because that chromatic is "unnatural"; it is the matter of adequate notation.

Now it's time to make an important concession. The demand of consistent alteration system for every "remote" musical culture is unnecessary rigorous. Moreover, the consequent alteration, so closely connected with the idea of transposition, is not suggested by any of 5 classical metavoli, mentioned in ancient Greek sources. They may be good for counterpoint but not for monodic or heterophonic cultures. Transposition, or, implied transposition, can quickly loosen melodic formulas, since each degree may enter each position of the system, and consequently each of 7 modes. That destroys the individual differences between the modes – for example Lydian can be seen as Ionian "from the 4th degree" or as Ionian with the augmented fourth.

But nevertheless – the possibility of 2-dimensional representation and exploration of its formalisms is useful. It would be impossible without it to compare different consistent alteration systems and to verify the concept of "attraction".

PROPAEDEUTICS

In order to grant the pitch symbols more general meaning than they have in "western" notation one needs some formal yet spacious framework. And here we are again by the question of approximating. No notation can render melodies wholly adequately. Yet, the choice can be made between different approximations. Following F. Raudonikas[10] we define the mode technically in Aristoxenean sense but without indication of concrete intervals. So, the mode is a combination of the tones that

1. can be rearranged in the gapless sequence iterating given interval A (*generative* parameter) with
2. the help of transfers iterating given interval B (*recurrent* parameter[11]), whereby this interval B is treated as an ambit of resulting mode.

In case of Aristoxenos the sequence is "made out of" perfect fifths, the recurrent parameter is an octave. In fact – to obtain the mode we exercise the octave-transfers of the degrees, obtained in a perfect-fifth-sequence. These octave-transfers place the mentioned degrees within the boundaries of octave.

Some other examples: diatonic tetrachord is the mode, as it suggests the major-second-evolving "within" the perfect fourth. Diatonic pentachord is also the mode. Chromatic modes also fall within the scope of this definition, some of them can be realized even on the usual keyboard. Chromatic tetrachord is the perfect fourth "filled up" with two minor seconds; chromatic pentachord requires 3 minor thirds (one might be "fifth-transferred") and perfect fifth as ambit.

Let's have a look on Fig. 6. Light triangles above are diatonic tetrachords. In the bottom of the figure they construct the "Myxolydian" mode from Fig. 2 (see the black line of tonica, constraining two light and two dark triangles).

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[10] Ravdonikas F. V., *Muzykal'nyj sintaksis*, 2-e izdaniye, Saint-Petersburg, Earlymusic Publishing House, 2007, p. 115–153.

[11] Recurrent – running or turning back in a direction opposite to a former course – Merriam-Webster Dictionary. Hence the new obtained degrees have to be "brought back", *i.e.* to be "octave-transferred" in the direction opposite to that of generating sequence (e.g. sequence of perfect fifths).

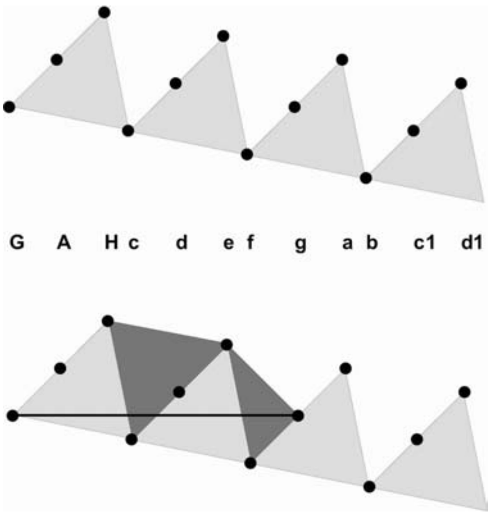

Fig. 6

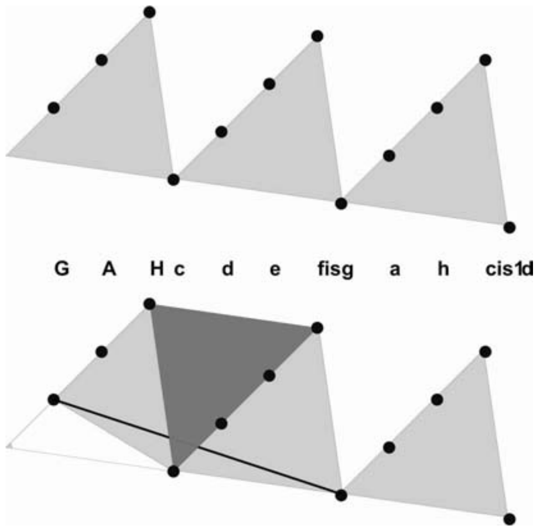

Fig. 7

Fig. 7 shows the same for the diatonic pentachords and shows the constructing of the "Ionian" mode, (already presented on the Fig. 4). Both Figs. (7-8) can be played on piano. Yet the total number of modes – chromatic and diatonic – rendered with trivial keyboard, is small. In fact – many modes being currently in use – like harmonic and melodic minors, or common keyboard-interpretations of "gypsy" ("Hungarian") modes – are no modes according to our definitions (they can't be reorganized into the gapless sequence of any interval). So, they look like *imitations* in respect of some musical realities that can never be *rendered* correctly with the help of trivial keyboard.

The problem of natural intervals is on the first glance unsolvable – no natural interval can close the evolving circle, no perfect fourths (4/3), no perfect fifths (3/2), no natural major thirds (5/4) etc. Yet, the character of the mode, its physiognomy may not totally depend on its "naturalness".[12] On the other hand – logarithmical representation can give *various* characters.

Before we proceed to the details we have to introduced the symbols and definitions used in the article. We speak of the *mode*, if the distribution of all its steps in regard to tonica-line is given. *Mode 2/4* means that two steps are below tonica-line, four steps are above tonica-line, *mode 3/3* means that there are three steps below and three steps above tonica line etc. *Degree* and *step* we use here as synonyms. Below and above tonica line have nothing to do with lower or higher pitch: if A (la) is higher in the *fifth-sequence* that D (re), all octave transfers of A are higher than all octave-transfers of D in this, and only this sequence, so they will be always above the tonica-line, and G (sol) will be for the same reason always below tonica line. The *system* of the musical tones is symbolized here with *ratio* (sometimes referred as *module*) of the generative parameter (**A**) to the recurrent parameter **(B)**. So, 5/12 is a system, where the octave is being "filled" with

[12] Rationality in aspect of pitch can be understood in two ways – first as an interval, *i.e.* ratio of two frequencies (4/3, 9/8 etc), second – as rational *logarithms* of the intervals ($2^{5/12}$, $2^{1/6}$ – to give the examples of "well-tempered" analogs of 4/3 and 9/8). These both rationalities can't be "rational at the same time" or, to use more traditional expression, they are incommensurable. Whichever rationality we choose, the other one would necessarily be an approximation. So, no equal division of octave would give a single rational interval (except of octave itself), *i.e.* an interval that can be presented as ratio of two commensurable frequencies – no perfect fourths (4/3), no perfect fifths (3/2), no other "superparticular" intervals (9/8, 10/9, 11/10 etc) are possible. And vice versa – no rational (in the above mentioned sense) interval can be a step of the equal (*i.e.* logarithmic) division of octave with finite number of degrees ("The circle of fifths can be never closed" as the school maxim states). Consequently no rational intervals can exactly render "tempered" scale, they can only approximate it. Such an approximation is a widespread practice, e.g. it is necessary for choir music a capella, where the singers need natural intervals. The process of approximation of natural intervals by their logarithms is mainly disregarded for its "artificiality". Yet, the merits of the opposite approximation have never been considered – logarithmical scale can be approximated by "natural" intervals. It is also clear that if the rational (in both senses) scale is given, it is not necessarily "bound" on staff. Yet, the opposite is not true – staff "suggests" logarithmical rationality just because the lines are set equidistantly. Hence the real importance (and novelty) of "staff" is, that it illustrates the independence of any interval of its actual pitch; not only intervals but all other segments (trichords, tetrachords ets) retain their "size" at any "height" (by the representation in frequencies this is apparently *not* the case, the "same" intervals would be extended by transfers: e.g. octaves – 220 Hz, 440 Hz, 880 Hz etc), yet this peculiarity of graphical representation was rather used by staff-notation than proposed by it. As it will be shown in the main text, this peculiarity may be used much more comprehensively.

tempered fourths, 7/12 is a system, where the octave is being "filled" with tempered fifths, etc. Unless otherwise specified, if the metrics is clear from the context, (e.g. the octave is always divided into 12 equal parts) all the ratios are "bound" on this metrics: 2/5 is the ratio of tempered major second to the tempered perfect fourth, **and not** two fifths of the octave to the octave itself. The *ratio (module)* doesn't depend on the *mode*, the *mode* doesn't depend on the *ratio*. Sometimes the ratio will be presented as an exponent: $2^{5/12}$ is both a tempered perfect fourth, and the system with a tempered perfect fourth as generative parameter, and an octave as recurrent parameter. "2" is a logarithm base here. If we have to present the system, where the generative parameter will be a tempered major second, and the recurrent parameter will be a tempered perfect fourth we will have to write $(2^{5/12})^{2/5}$ because this time "$2^{5/12}$" is the logarithm base, yet the expression $(2^{5/12})^{2/5}$ will be sometimes simplified to $2^{1/6}$. However strictly speaking $2^{1/6}$ is not quite correct, for this expression implies major second as a generative parameter and octave as the recurrent parameter, which is completely autonomous system, having nothing to do with diatonic octachords and tetrachords.

We start with three scales which comply with Aristoxenean definition of diatonic scale, yet are different from one another.

1) $A = 2^{5/12}$ $B = 2^1$ (trivial keyboard-diatonic)

2) $A = 2^{7/17}$ $B = 2^1$

3) $A = 2^{8/19}$ $B = 2^1$

These scales have individual characters, their representation in forms of rational logarithms allows comparison with other characters, not necessarily diatonic ones.

In the following Tables the respective frequencies (in Hz) of the relevant scales are given. The frequencies will help the curious reader to make the phonograms with the help of common sound editors, like Nero WaveEditor or Cool Edit and to compare his/her own hearing impression with those presented in the article. Table 1 presents the three above-listed diatonic scales. The modules are indicated in the leftmost column. The mode 2/4 is presented.

Table 1

Mode 2/4 in three different systems

5/12	***220***	***246.9***	***261.7***	***293.7***	329.7	349.2	392.1	440
7/17	***220***	***248.6***	***259***	***292.7***	330.8	344.5	389.4	440
8/19	220	245.4	264.1	294.6	328.7	353.5	394.4	440

The scale of the system 5/12 allows comparison of diatonic tetrachords ($A = 2^{1/6}$ $B = 2^{5/12}$ – normally we would say "A is the tempered major second, B is the tempered perfect fourth, containing two and five semitones respectively") and diatonic octachords ($A = 2^{5/12}$ $B = 2^1$ – five and twelve semitones respectively), to bring one of few examples that can be played on piano. (Note: $2^1 = 2^{12/12} = 2^{17/17}$ etc. = the octave). Yet, both other systems – 7/17 and 8/19 - in spite of their bright and distinguishable characters can't be "mapped" on the keyboard-pattern one-to-one, because the "keyboard-octave" is divided into 12 semitones – not more and not less. (Piano may be always retuned, but it wouldn't be possible to fit these scales to the pattern of the keyboard in consistent way).

The considerations of performance practice require approximation of logarithmical intervals by the natural (rendered with ratios of frequencies) ones. For example, the above mentioned diatonic tetrachord with $A = 2^{1/6}$ $B = 2^{5/12}$, illustrated in Fig. 6, sounds more beautifully in its "natural correction" ($A = 9/8$ $B = 4/3$), yet the characters of both tetrachords are similar. On the opposite – the tetrachord, "extracted" from the octachord $A = 2^{7/17}$ $B = 2^1$, which has $A = 2^{3/17}$ $B = 2^{7/17}$, is "logarithmic", yet the character is different. (On the Table 1 the respective frequencies are given in bold italics). The "demarcation-line" in musical characters between $A = 2^{1/6}$ $B = 2^{5/12}$ and $A = 2^{3/17}$ $B = 2^{7/17}$ is very clear here. The best "natural" approximation for $A = 2^{3/17}$ $B = 2^{7/17}$ seems to be "diatonica tonayon" of Ptolemeus, or something like $A = 8/7$ $B = 4/3$.

If we take an octachord $A = 2^{5/17}$ $B = 2^1$, the "extracted" tetrachord with $A = 2^{2/17}$ $B = 2^{7/17}$ can be "naturally" approximated with $A = 12/11$ $B = 4/3$. It sounds also very specifically. Yet (because of the common denominator) the two last mentioned tetrachords (*i.e.* $A = 2^{3/17}$ $B = 2^{7/17}$ and $A = 2^{2/17}$ $B = 2^{7/17}$) can turn into one another by a shift of a single degree to the 1/17 of octave! See Table 2, the only "varying" degree of the lower tetrachord (columns 3–6) is given in bold italics in both systems; the rest of mentioned tetrachord is identical in both systems:

Table 2

Mode 2/4 module 5/17 vs Mode 4/2 module 7/17

5/17	2/4	220	248.6	***269.7***	292.7	330.8	358.9	405.5	440
7/17	4/2	220	248.6	***280.9***	292.7	330.8	373.8	389.4	440

The "natural" presentations (8/7 : 4/3 vs 12/11 : 4/3) don't reflect this beautiful ambiguity, whereas logarithmical picture is very expressive – it is enough to compare all three numerators ($A = 2^{2/17}$, $A = 2^{3/17}$ and $B = 2^{7/17}$) and build alternative tetrachords: 3 + 2 + 2 = 7 ($A = 2^{2/17}$, $B = 2^{7/17}$) vs 3 + 3 + 1= 7 ($A = 2^{3/17}$, $B = 2^{7/17}$). Mode 2/4 of the system 5/17 represents the division:

3 + 2 + 2 = 7 ($A = 2^{2/17}$, $B = 2^{7/17}$) ,

Module 7/17 mode 4/2 represents the division:

3 + 3 + 1= 7 ($A = 2^{3/17}$, $B = 2^{7/17}$)

Such examples clearly show the disjunctive role of logarithmically equidistant divisions of the ambit – in fact 2/7 is less than 1/3, 3/7 is more than 1/3. (Since 2 : 7 = 2/17 : 7/17 we omit here the denominator 17). So 1/3 acts as disjunctive section or "anti-attractor". In this context it is interesting to contemplate about some other possible meanings of terms "paramese" and "trite". They may be seen not just as names of fixed or movable degrees, but also as parts of an ambit of the mode ("close-to-half" and "one-third"). In this meaning these names are not being bound on a *single* particular scale, but can characterize different scales according to behavior of these scales in relation to their parts (attraction-phenomena). More important is that the *half* of the ambit as a "distinctor" of genera (diatonic = sum of 2 steps is *more* than the half of ambit, chromatic = sum of 2 steps is *less* than the half of ambit) is not so unique in its distinctive function – 1/3, 1/4 and others can be distinctive, though with less expressed contrasts. Even if this induction looks sophisticated it is clear that only the logarithmical picture allows the comparison, the revealing of structural analogy, common combinatorial provenance and description of attraction-repulsion effects.

The continuum of different two-dimensional consistent alteration systems – even if the recurrent parameter and number of degrees are fixed – is infinite, because each ambit can be potentially divided into arbitrarily large number of equal parts. Yet, the above discussed circumstances allow speaking about affinities that make the amount of characters at least observable. If we schematically arrange two-dimensional systems one upon another like sheets of paper in a packet – according to the increasing ratio of generative parameter to recurrent parameter (generally – from 1/x to x/x, for every natural x) – while recurrent parameter itself remains constant – the whole continuum of systems will dissipate into *layers*. The decisive circumstance, responsible for forming of the layers, is already touched upon – that is a participation of equidistant ambit divisions where number of parts is less than denominators of scales in question (e.g., 1/3 in comparison with 2/7, or 2/5 in comparison with 5/12). These divisions are unavoidable in the construction, because the construction is a continuum – when moving from 2/7 towards 2/5 it is impossible to "skip" 1/3. These divisions act disjunctively. Generally progressive evolving of any logarithmical scale has a moment of degeneration (=scale becomes equidistant). It can be proven that the modes according to our generalized Aristoxenean definition never contain more than 3 different neighbor intervals.

Such a scrutiny may arouse definite mistrust, but it may be seen as a basis for studying characteristics of different scales as well. Now it's time to make use of two-dimensional diagrams.

CONSEQUENCES

On Fig. 8 two diatonic scales are presented.

Polygon on the top of Fig. 8 represents trivial diatonic mode with $A = 2^{5/12}$ and $B = 2^1$. Generative parameter is the tempered perfect fourth, recurrent parameter is octave. So, the whole underlying system would be marked as 5/12. Polygon in the bottom of Fig. 8 is the same mode (4 degrees above, 2 degrees below tonic), but in the system 5/13, *i.e.* $A = 2^{5/13}$ $B = 2^1$. They sound similarly (except of the very broad step between 7th and 8th degrees of the 5/13 module scale that sounds rather like a "leap") and still belong to different (yet adjacent) layers. Disjunctive section is 2/5 ($A = 2^{2/5}$ $B = 2^1$). How would this circumstance affect the notation?

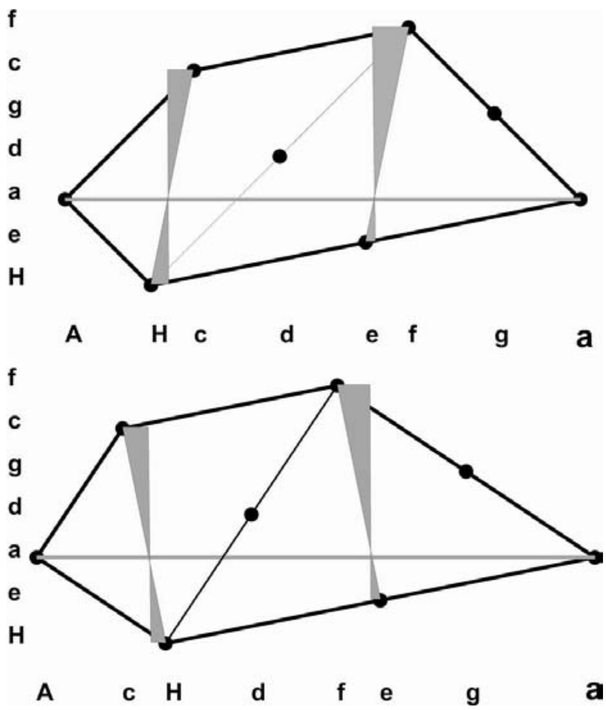

Fig. 8

Table 3

Mode 2/4: module 5/12 vs module 5/13

5/12	220	246.9	261.7	293.7	329.7	349.2	392.1	440
5/13	220	244.7	258.1	287.2	319.5	337.1	375	440

Before answering this question I shall explain the actual reason of it. The probability that Znamenny chant would primordially belong to the same layer as the scale of trivial keyboard is small, yet the authors of bilinguals of 17^{th} century should have found the transition more or less tolerable. That gives reason to look for the characters that *resemble* the Guidonian, being however represented by their own systems, which don't *deviate* from the Guidonian, and even disclosure the Guidonian system itself as a certain deviation. Here we have such a system (5/13). Both presented scales are built nearly with the same interval. So, let us numerate both sequences with the same symbols. We can choose for example the conventional letters for ascending fourths. In Fig. 8, both sequences of fourths are shown vertically on the left. And now look at the result, represented in horizontal lines of letters. The letters showing semitones look interchanged, though the hearing impression was that the conjunct tetrachords (*i.e.* from "A" to "g" through all other degrees) in 5/13 sound just a bit narrower and an upper tone "g-a" sounds essentially broader. This interchanging results from the both negative degrees -1 and -2 having shifted their position in relation to positive degrees 3 and 4. Corresponding deviations are shown with triangles, attached to from above and from below to some points on the tonica-line. This tonica-line can be seen at the same moment exactly as the mentioned disjunctive "anti-attractor", *i.e.* the scale with $A = 2^{2/5}$ $B = 2^1$. It is easy to see, that the triangles "grow" from this line up and down, exactly from the points of division of ambit (=recurrent parameter) into 5 parts – *i.e.* from the points 1/5 and 3/5. Exactly these points may be considered as the "norm" both scales deviate from. Of course, all that gives *no* privileges to Guidonian scale.

Figs. from 9 to15 show that negligible differences in hearing impression can have grave consequences for notation. The letters and notes seem to be totally confused. Yet, they are correct. For all of them I've retained the "vertical" denomination ("solmisation") from the left side of Fig. 8, whereas "horizontal" denominations depend entirely on the always changing value of interval A (*i.e.* of the generative parameter, in case of Figs. 9–15 it is 5/13). Flats have "descendive" meaning, but the reference for descending-ascending is the line of tonic: "high" and "low" frequencies in common sense are lying *to the right and to the left* of our 2-dim. diagrams, and not "in the top" and "in the bottom" of it and thus have nothing to do with flats and sharps.

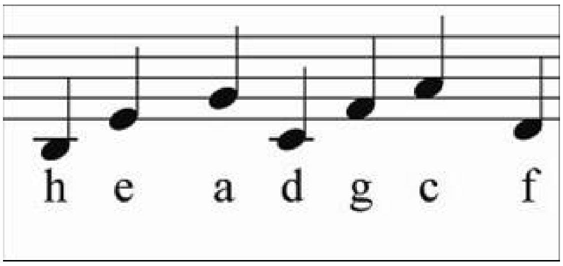

Fig. 9

Table 4

The unfolding of the module 5/13 system, first 6 positive steps (= mode 0/6)

220	287.2	375	244.7	319.5	417.2	272.3

Fig. 9 shows the evolving (=generative fourths-sequence with corresponding octave-transfers) of the mode, where all 6 fourths-steps lie above tonic-line. Since we have adopted only 7 "natural" degrees, and all of them have been already used up in this positive evolving of Fig. 9 (generally that depends entirely on the choice of the degree-name for tonic), the evolving in the negative direction requires flats, as illustrated in Fig. 10.

Fig. 10

Table 5

The unfolding of the module 5/13 system, first 6 negative steps (= mode 6/0)

440	337.1	258.1	395.5	302.9	232.1	355.5

The next four pictures show different modes where the sequence of fourths is rearranged to form scale-representations of modes. They differ only in distribution of the degrees in regard to tonic-line:

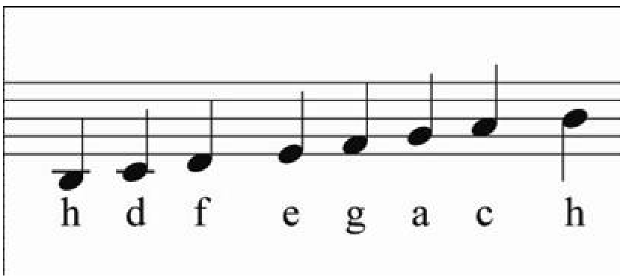

Fig. 11 – all degrees are above tonic.

Fig. 12 – 5 degrees are above, 1 degree is below.

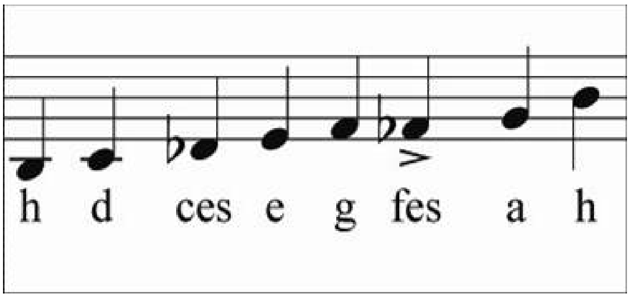

Fig. 13 – 4 degrees above, 2 degrees below.

Fig. 14 – 3 above, 3 below.

The sense of alteration is clear – “fes” of Fig. 12 substitutes “f” of figure 11, “ces” and “fes” of Fig. 13 substitutes “c” and “f” of Fig. 11 and so on.

Note:

In trivial diatonic scales (5/12 or 7/12) the “natural” and “altered” forms of degree are at a distance of 1/12 of octave. The connotation “to be nearly the same” seems to be natural for the notion of “altered degree” as soon as 1/12 of octave is a shortest possible distance in this system. Yet this connotation is artificial, and thereby a very cute one: 7th fifth (if count off from the “root”) and also 7th fourth lie close to the multiples of 12:

5 x 7 – 3 x 12 = -1

7 x 7 – 4 x 12 = 1

That is a choice of inventor of our keyboard. 12 semitones presuppose 7 “natural” degrees to achieve the maximal “proximity” of natural and altered degree. 7th fifth of evolving requires 8th *name* of the degree, or alteration of the first name. It *coincides* with “1/12-deviation”, nothing more. By 5/12 not only 7th fifth (fourth) but also 5th fifth (fourth) gives a semitone:

5 x 5 – 2 x 12 = 1

7 x 5 – 3 x 12 = -1. That is sometimes explained as disregard towards distinction of “natural” and “chromatic” (artificial) semitone, clearly demonstrated by trivial keyboard. Analogous considerations show that 5/13-system would require either 5 or 8 natural degrees to achieve this semitone-proximity, 5/17-system it would require either 7 or 10 natural degrees and so on. In our examples here (5/13) we have only 7 natural degrees and thus distance between f and fes is either 8 “semitones” or 5 “semitones” – which is uncomfortable, but correct.

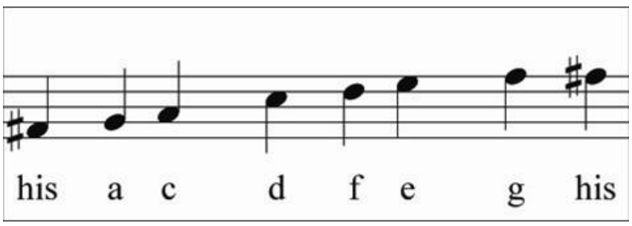

Fig. 15

Fig. 15 is an example of the discussed connection between nomenclatures of degrees and modes. “Tonica” can’t be natural and altered at the same time. In case of 5/12 this awkward circumstance is masked by the retained positions of other degrees (H-c-d-e-f-g-a-h (6 fifths-steps above tonic) vs B-c-d-e-f-g-a-b (6 fifths-steps below tonic)). But in case of 5/13 and in the majority of other systems that doesn’t work anymore. Natural and altered forms are not so “symmetrical” towards other degrees.

Table 6

Modes of the system 5/13

0/6	220	244.7	272.3	287.2	319.5	375	417.2	440
1/5	220	244.7	287.2	319.5	337.1	375	417.2	440
2/4	220	244.7	258.1	287.2	319.5	337.1	375	440
3/3	220	244.7	258.1	287.2	337.1	375	395.5	440

4/2	220	258.1	287.2	302.9	337.1	375	395.5	440
5/1	220	232.1	258.1	287.2	302.9	337.1	395.5	440
6/0	220	232.1	258.1	302.9	337.1	355.5	395.5	440

In the next pair of scales the precondition of the melismatical technique can be traced.

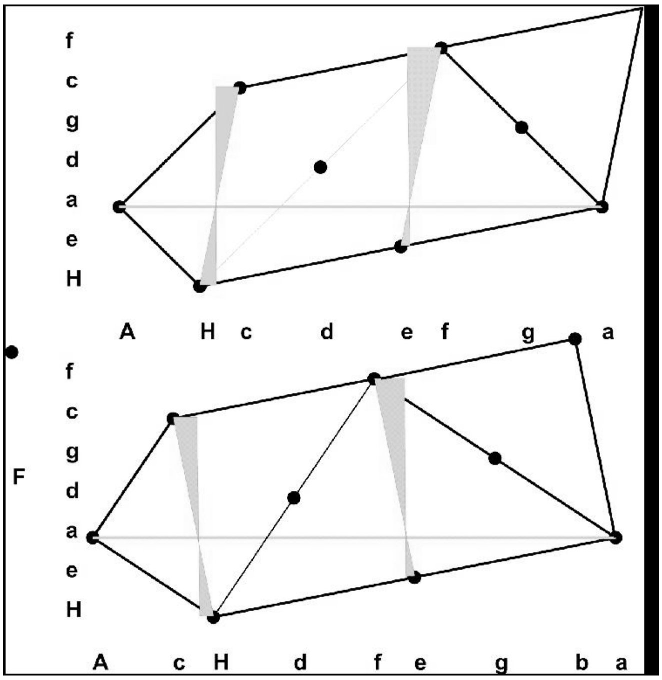

Fig. 16

Table 7

Nonachords. Mode 2/4 of the system 5/12 is being added a minor second above the upper tonica. But the upper line of the Table can be also seen as the mode 2/5 with a minor none as an ambit (e.g. H-c) The lower line is the mode 2/5 of the system 5/13 with perfect octave as an ambit (H –h)

5/12	220	246.9	261.7	293.7	329.7	349.2	392.1	440	466.2
5/13	220	244.7	258.1	287.2	319.5	337.1	375	417.2	440

Fig. 16 gives the same scales as Fig. 8, but with additional step. Thus the scales are nonachords – 5 degrees above tonica-line, 2 degrees below it (5 above + 2 below + 2 “tonical” degrees = 9). The ratio of one ambit to another is 13/12 (i.e. ratio of minor none to perfect octave), their structure is the same. The upper left vertex of the upper polygon on Fig. 16 (the highest point) can be seen as b (si-bemol). This b can be interpreted however also as a new upper border of the ambit, “stretching” the whole ambit to 13/12 of origin value. In this case we would have no more $A = 2^{5/12}$ $B = 2^{12/12}$ + semitone, but $A = 2^{5/12}$ $B = 2^{13/12}$. The same ratio of generative and recurrent parameters (5 through 13) we have in the nonachord in the bottom of Fig. 16, $A = 2^{5/13}$ $B = 2^{13/13}$. This interpretation ambiguity is characteristic for the pairs of scales with the same generative parameter, and with the ambits close to one another but not identical, such as 5/17 – 5/18 or 3/10 – 3/11 (for both pairs the disjunctive section is 2/7) – the denominator shan’t necessary be larger than 12. That proves the distinction between micro- and macro-intervals to be not very accurate convention.

It is not clear whether the theorists of that time noticed these ambiguities. But they would certainly have felt them. The nature of the melos will always strive against the mortifying effects of notation. So, we have seen an example of this uncertainty – neither new degree, nor modification of the old one, neither embellishment note nor new tonic. It illustrates the process of “winding” of the melos from one mode to another. This winding is often reflected in the names of melodic formulas of Znamenny chant and even as verbal expression of melodic formation. It is also an important example to show *deviation* from one norm as necessary *approaching* to another one.

It has been shown only a small part of metamorphoses the nomenclature has to undergo whereas the characters remain akin, or can be estimated as “slightly deviating”. I’d like you to hear the already mentioned pair 5/17-5/18.

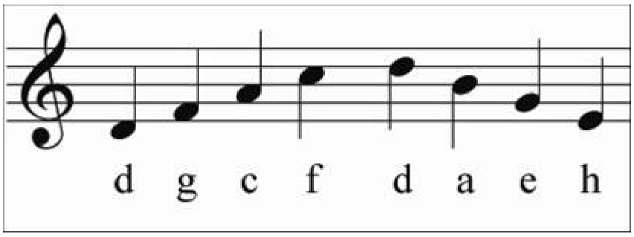

Fig. 17

Fig. 17 shoes evolving of the mode 3/3 (three steps above tonic, three steps below tonic in both systems).

Table 8

Evolving of the systems 5/17 and 5/18 (3 steps in positive and 3 steps in negative directions, giving the mode 3/3)

5/17	220	269.7	330.8	405.5	440	358.9	292.7	238.7
5/18	220	266.7	323.3	392.1	440	363	299.4	246.9

Fig. 18 shows the modes (from top to bottom of the Fig.) 3/3, 2/4, 1/5 and 0/6 for both systems (5/17 and 5/18). The characters of the modes of both systems are not as contrasting as of those already discussed. 5/17 is more severe, 5/18 is softer. The modal differences are subtle; so many melodic formulas can be sung in both systems and in different modes of each system. The mode with Phrygian lower tetrachord may have a “major” upper tetrachord, which makes traditional attribution questionable. The Phrygian cadence, characterizing so many fita’s and also many popevkas (regular cadence formula), would have in these systems three different variants, which are unavoidably “overlooked” by the traditional notation. That explains, to some extent, why the Phrygian cadence has been used in many different “glasy” (=modes in authentic Russian terminology).

Fig. 18

Table 9

Modes of the system 5/17

0/6	220	248.6	269.7	304.9	330.8	373.8	405.5	440
1/5	220	248.6	269.7	304.9	330.8	358.9	405.5	440
2/4	220	248.6	269.7	292.7	330.8	358.9	405.5	440
3/3	220	238.7	269.7	292.7	330.8	358.9	405.5	440

Table 10

Modes of the system 5/18

0/6	220	237.6	266.7	288.1	323.3	349.2	392.1	440
1/5	220	237.6	266.7	288.1	323.3	349.2	363	440
2/4	220	237.6	266.7	299.4	323.3	349.2	363	440
3/3	220	246.9	266.7	299.4	323.3	349.2	363	440

Both systems are different yet they may use the same notation consistently. The only difference is that in one case sharp will signify "deviation" of degree to discantus, in other case – deviation to bass. Since discantus and bass don't coincide with top and bottom of diagram, notation remains correct in both cases. It shows that notation sometimes is not only too specific "to know" about affinities, but also too ambiguous, "to know" about differences.

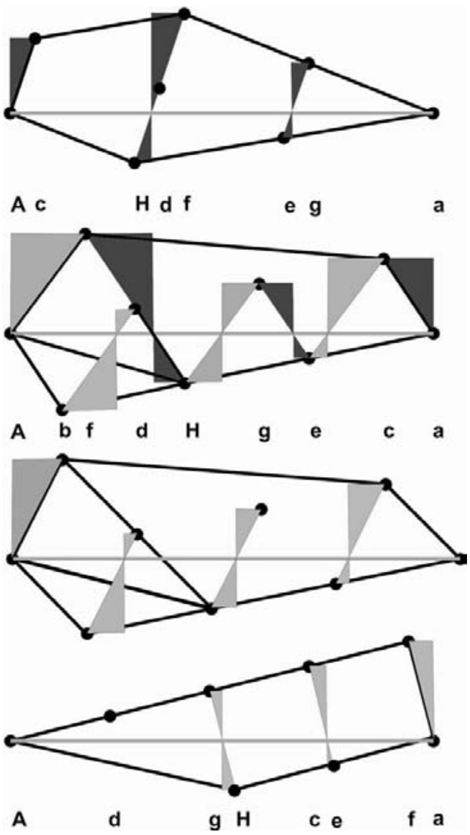

Fig. 19

As Fig. 19 shows, the deviation from one isometric section (equidistant division of the ambit with *disjunctive effect*) can be at the same time the approaching to another isometric section (another equidistant division, which is denser that the first one and therefore exercises the *attractions-effect*). Different deviations from different isometrics are shown with different orientations of the triangles.

The scales on the Table 1 have illustrated concurring regular divisions of the scale, which correspond the features of the scale of Znamenny chant, as presented in the treatise of Mezenets. These features suggest regular iterations of two intervals of the same size and one interval of another size throughout the whole scale. As it follows from the above-considered circumstances the scale remains diatonic by *different* values of intervals. Only 3 combinations are being used with the relations of broader to narrower interval 2:1 (for A = $2^{5/12}$), 3:2 (A = $2^{8/19}$) and 3:1 (A = $2^{7/17}$). Even this modest range of relations implies division of the octave into 12, 17 and 19 parts. Yet, thereby 2 further objectives have been considered:

1. The first and the eighth degrees of the scale give an octave.

2. Sequence of two broader and one narrower intervals have been repeated, and not that of two narrower and one bigger intervals.

Needless to say, that neither of these both objectives is directly, or indirectly implied by the treatise of Mezenets and, consequently, by Znamenny notation. Moreover, the phonograms of the scales of the systems 5/17 and 5/18 clearly show that violation of these two objectives by no means destroys the diatonic character of the scale – if we have to assume that the scale *was* diatonic. If we neglect these two objectives, the variety of possible scales would be much more impressive. So, Mezenets would have reasons not to restrict the scale to just *one* variant of octave-division: as an opponent of staff-notation he would appreciate the possibilities of intervallic flexibility. As we have already seen, given such diversity, it is unjustified to claim any unified orthography, in particular, the systems of cinnabar letters would inevitably contradict each other if applied to different scales from this set. Thus the usage of sipavyje, kryzhevyje etc. pomety (=different generations of "pitch-refining" letters) can be commented upon without any recurring to transpositions.[13] Very symptomatic is the fact that characteristic formulas of the modes have never been furnished with cinnabar letters. What could be the reason for that? If we consider Azbooka of Mezenets as an instructive book, it would be justifiable to expect the opposite – exactly these, the most characteristic features in the chant should have been subject to detailed deciphering. Stenography is not the only reason for restraining oneself from that – many formulaic cadencies are not really stenographic. Maybe, the freedom to interpret textual regularities had been felt more dangerous, than freedom of different melodic realizations of the same ideographic outline.

CONCLUSION

The alternative of the empirical deviations assessment in music, and generally in the art can be described as follows: either one conceives the practice as necessary deviations from some given apriori system – one thinks, e.g. on Vitruvius, who needed his *temperaturae* (which he interpreted as deviations from the mathematic proportions) to animate the calculated forms – or one has to humbly reconcile oneself with an inexplicable fact of practice and approach it with simplifying schemes. The opposition of these two options can be weakened by the idea of *different* possible systems and *different* ways to approximate them. The presented study shows that in general no immediate results are to be expected. Yet, this polyvalent approach gives variety, choice and freedom, which is nice in and of itself, even if these things are sometimes difficult to endure.

Although this may seem a paradox, all exact science is dominated by the idea of approximation.
(Bertrand Russell)

[13] About the treatise of Mezenets and different kinds of pomety see: Z.M. Gusseynova, *"Izveshcheniye" Aleksandra Mezentsa i teoriya muzyki XVII veka*, St. Petersburg, 1995.